Menoufeya University
College of Engineering
Civil Engineering Dept.

Minufiya University

Finite Element
June 2015
Time 3.0 Hours

OPEN BBOK EXAM

Question (1)

a- If $S^{(l)}$ and $S^{(u)}$ denote the perimeters of the inscribed and circumscribed polygons, respectively, as shown in Figure 1.3, prove that :

$$S^{(l)} \leq S \leq S^{(u)}$$

b- Generate the finite element mesh for the two-dimensional object shown in Figure 2.37 using the quadtree method.

c- The quadratic interpolation function of a one-dimensional element with three nodes is given by :

$$\varphi(x) = \alpha_1 + \alpha_2 x + \alpha_3 x^2$$

If the x coordinates of nodes 1,2, and 3 are given by 1, 3, and 5, respectively, determine the matrices $[\eta]$, $[\eta]^{-1}$, and [N] of Eqs.(3.17), (3.18), and (3.20).

Question (2)

a- Consider the shape functions, $N_i(x)$, $N_j(x)$, and $N_k(x)$, corresponding to the nodes *I, j, and k* of the one-dimensional quadratic element described before. Show that the shape function corresponding to a particular node *i (j or k)* has a value of one at node *i (j or k)* and zero at the other two nodes *j (k or i)* and *k (i or j)*.

b- The Cartesian (global) coordinates of the corner nodes of a quadrilateral element are given by (0,-1), (-2,3), (2,4), and (5,3). Find the coordinates transformation between the global and local (natural) coordinates. Using this, determine the Cartesian coordinates of the point defined by *(r,s)=(0.5,0.5)* in the global coordinate system.

c- Determine the Jacobian matrix for the quadrilateral element defined in problem (2-b). Evaluate the Jacobian matrix at the point *(r,s)=(0.5,0.5)* .

Question (3)

a- Evaluate the integral :

$$I = \int_{-1}^{1} ((a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4) dx$$

Using the following methods and compare the results:

i- Two-point Gauss integration

ii- Analytical integration.

b- Evaluate the partial derivatives $(\partial N_i/\partial x)$ and $(\partial N_i/\partial y)$ of the quadrilateral element shown in Figure 4.30 at the point $(r = 1/2\ , s = 1/2)$ assuming that the scalar field variable φ is approximated by quadratic interpolation model.

c- The deflection of a beam on an elastic foundation is governed by the equation $(d^4w/dx^4 + w = 1,$ where x ,and w are dimensionless quantities. The boundary conditions for simply supported beam are given by transverse deflection $= w = 0$ and bending moment $= (d^2w/dx^2) = 0$ .By taking a two-term trial solution as $w(x)=C_1 f_1(x) + C_2 f_2(x)$ with $f_1(x) = \sin\pi x$ and $f_2(x) = \sin 3\pi x$, find the solution of the problem using the Galerkin method.

Question (4)

a- The cantilever beam shown in Figure 5.10 is subjected to a uniform load w per unit length. Assuming the deflection as

$$\varphi(x) = c_1 \sin\frac{\pi x}{2l} + c_2 \sin\frac{3\pi x}{2l}$$

Determine the constants c_1 and c_2 using the Rayleigh-Ritz method.

b- Consider the differential equation

$$\frac{d^2\varphi}{dx^2} + 400\,x^2 = 0 \qquad 0 \le x \le 1$$

With boundary conditions $\varphi(0) = 0$, $\varphi(1) = 0$

The functional of the problem to be extremized is given by

$$I = \int_0^1 \left\{ -\frac{1}{2}\left[\frac{d\varphi}{dx}\right]^2 + 400x^2\varphi \right\} dx$$

Find the solution of the problem using the Rayleigh-Ritz method using a one term solution as $\varphi(0) = c_1 x(1-x)$

c- If the elements characteristic matrix of an element in the finite element grid shown in Figure 6.4 is given by

$$[K^{(e)}] = \begin{bmatrix} 3 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 3 & 1 \\ 1 & 1 & 1 & 3 \end{bmatrix}$$

Find the overall or system characteristic matrix after applying the boundary conditions $\varphi_i = 0$, i=11-15. Can the bandwidth be reduced by renumbering the nodes ?

Question (5)

a- Find the eigenvalues and eigenvectors of the following matrix using the Jacobi method :

$$[A]=\begin{bmatrix}3 & 2 & 1\\ 2 & 2 & 1\\ 1 & 1 & 1\end{bmatrix}$$

b- Solve the following system of equations using the Cholesky decomposition method using (i) $[L][L]^T$ decomposition and (ii) $[U]^T[U]$ decomposition:

$$5x_1 + 3x_2 + x_3 = 14$$
$$3x_1 + 6x_2 + 2x_3 = 21$$
$$x_1 + 2x_2 + 3x_3 = 14$$

c- Determine whether the following state of strain is physically realizable :

$\varepsilon_{xx}=c(x^2+y^2)$, $\varepsilon_{yy}=cy^2$, $\varepsilon_{xy} = 2cxy$, $\varepsilon_{zz}= \varepsilon_{yz} = \varepsilon_{zx} = 0$

Where c is constant.

d- Consider the following state of stress and strain :

$\sigma_{xx}= x^2$, $\sigma_{yy}=y^2$, $\varepsilon_{xy} = -2xy$, $\sigma_{zz} = \varepsilon_{xz} = \varepsilon_{yz}=0$

Determine whether the equilibrium equations are satisfied.

Question (6)

a- Consider the following condition :

$\varepsilon_{xx}=c_1x$, $\varepsilon_{yy}=c_2$, $\varepsilon_{zz}= c_3x + c_4y + c_5$, $\varepsilon_{yz} = \varepsilon_{zx} = 0$

Determine whether the compatibility equations are satisfied.

b- A beam is fixed at one end , supported by cable at the other end, and subjected to a uniformly distributed load of 50 lb/in. as shown in Figure 9.26.

i- Derive the finite element equilibrium equations of the system by using one element for the beam and one element for the cable.

ii- Find the displacement of node 2.

iii- Find the stress distribution in the beam.

iv- Find the stress distribution in the cable.

c- A water tank of weight W is supported by a hollow circular steel column of inner diameter d, wall thickness t, and height h. The wind pressure acting on the column can be assumed to vary linearly from 0 to p_{max}, as shown in Figure 9.32. Find the bending stress induced in the column under the loads using one-beam element

$W=15,000$ lb, $h=30$ ft , $d= 2$ft , $t= 2$ in , $p_{max} = 200$ psi

d- Determine the stress distribution in the two members of the frame shown in Figure 9.28. Use on finite element for each member of the frame.